Matematisk – lathund

Räknelagar

$a + b = b + a$ *(kommutativa lagen under addition)*

$a \cdot b = b \cdot a$ *(kommutativa lagen under multiplikation)*

$(a + b) + c = a + (b + c)$ *(associativa lagen under addition)*

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ *(associativa lagen under multiplikation)*

$a \cdot (b + c) = a \cdot b + a \cdot c$ *(distributiva lagen)*

$a + c = b + c \Leftrightarrow a = b$ *(annulleringslagen under addition)*

$a \cdot c = b \cdot c \Leftrightarrow a = b \quad om\ c \neq 0$ *(annulleringslagen under multiplikation)*

Bråkregler

$$a \cdot \frac{b}{c} = \frac{a}{1} \cdot \frac{b}{c} = \frac{ab}{c} \qquad c \neq 0$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \qquad b \neq 0, d \neq 0$$

$$\frac{a}{b} \Big/ \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a}{b}\frac{d}{c} = \frac{ad}{bc} \qquad b \neq 0, c \neq 0, d \neq 0$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd} \qquad b \neq 0, d \neq 0$$

Parentesregler

$$a + (-b) = a - b$$
$$a \cdot b = ab$$
$$a - (-b) = a + b$$
$$a \cdot (-b) = a(-b) = -ab$$
$$(-a) \cdot (-b) = (-a)(-b) = ab$$

Algebra

Låt $a, b, c \in \mathbb{R}$ och $m, n \in \mathbb{Z}$.

$(a + b)^2 = a^2 + 2ab + b^2$ *(första kvadreringsregeln)*

$(a - b)^2 = a^2 - 2ab + b^2$ *(andra kvadreringsregeln)*

$$(a+b)(a-b) = a^2 - b^2$$ *(konjugatregeln)*

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Förstagradsekvationen

$$ax + b = 0 \qquad a \neq 0$$

$$x = -\frac{b}{a}$$

Kvadraträtter

$$a \text{ och } b \geq 0,\ c > 0$$

$$\sqrt{a} \cdot \sqrt{a} = a$$

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

$$c\sqrt{a} = \sqrt{c^2 a}$$

$$\frac{\sqrt{a}}{\sqrt{c}} = \sqrt{\frac{a}{c}}$$

$$\frac{a}{\sqrt{c}} = \frac{a\sqrt{c}}{c}$$

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

$$a\sqrt[n]{b} = \sqrt[n]{a^n b}$$

Potensregler

$$1^n = 1$$

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n}$$

$$a^1 = a$$

$$a^0 = 1 \qquad a \neq 0$$

$$a^{-n} = \frac{1}{a^n} \qquad a \neq 0$$

$$a^{1/2} = \sqrt{a}$$

$$a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m} \quad m,\ n\ > 0$$

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad a \neq 0$$

$$(ab)^m = a^m \cdot b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \qquad b \neq 0$$

$$(a^m)^n = a^{m \cdot n} = (a^n)^m$$